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ABSTRACT

The workbook is intended to help the vocational high school student understand and gain competence in working with fractions. The exercises provide practice in reducing and changing fractions, multiplying, dividing, finding a least common denominator and equivalent fractions, adding and subtracting. The types of fractions are also defined. Quizzes and problems with practical applications are utilized in the text to supplement the arithmetic problems. (RG)

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STATE OF NEW JERSEY
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Fractions and Their Applications

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State of New Jersey
Department of Education
Division of Vocational Education

FRACTIONS AND THEIR APPLICATIONS - A MATH PRACTICE BOOKLET

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Objectives

This booklet attempts to clarify the meaning of fractions and simplify the various operations as they involve fractions. Practice is provided as each new concept is learned.

By the end of this booklet, the student should be able to

1. Understand what a fraction is.
2. Reduce fractions to lowest terms.
3. Change an improper fraction to a mixed or whole number.
4. Change a mixed number to an improper fraction.
5. Multiply fractions.
6. Divide fractions.
7. Find a least common denominator and equivalent fractions.
8. Add fractions.
9. Subtract fractions – with and without borrowing.

TO THE STUDENT

Someday our country will do all its measuring and figuring by the metric system, which is based on decimals. Until that time comes, however, a great many of our measurements will involve fractions, and hence a great many of our math problems will involve fractions.

The cook and baker use fractions of cups, quarts, pounds, and so forth. The needle-trades worker uses fractions of a yard and of an inch. The carpenter, metalworker, plumber, auto mechanic, mason, welder – all need to work with fractional measurements. Indeed, there is scarcely an occupation that doesn't involve working with fractions at some point.

To take your place in the world of work without being held back by lack of skill in this area, you need to be familiar with fractions and to know how to work with them.

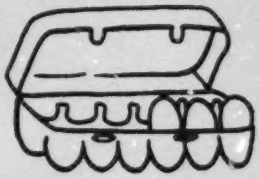
I – WHAT IS A FRACTION?

The term "fraction" means part of a whole. A fraction of a thing means that you are taking part of that thing. A fraction of a group means that you are taking part of that group.

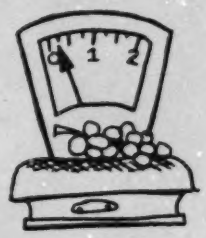
¼ of a pie

¼ of a cup

¼ of an hour



¼ of a dozen



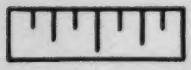
¼ of a pound

A fraction consists of two parts: the numerator and the denominator. The *numerator* is the upper number of the fraction, and the denominator is the lower number of the fraction.

Example: $\frac{1}{8}$ numerator / denominator

Before we go any further, let's be sure we really understand what fractions are. Take the fraction $\frac{3}{8}$.

The *denominator* gives the *name* of the fraction. In this case, whatever it is we are talking about is divided into *8* parts.

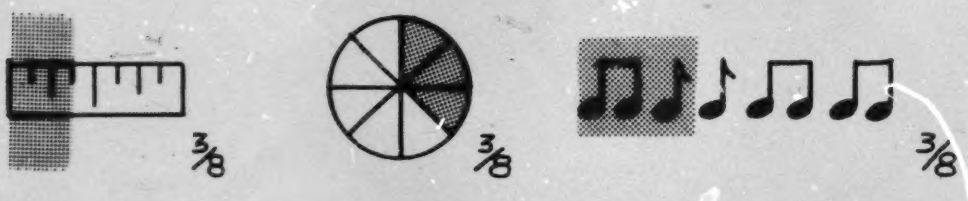


one inch

one pie

one whole note (in music)

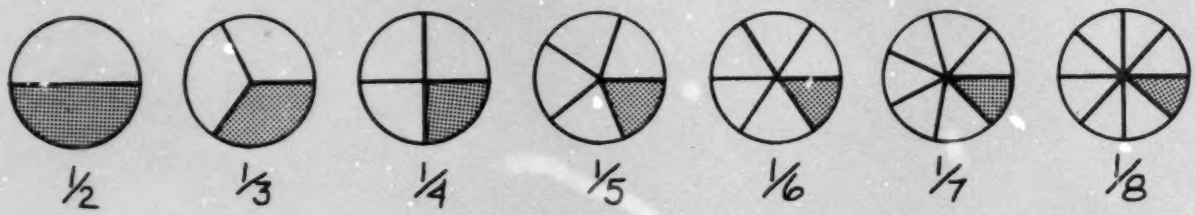
An inch, a pie, and a whole note in music have been divided into 8 parts. Each part is 1/8 of the whole.



Now we have counted out – *enumerated* or *num*bered – *3* of those parts; each shaded section represents 3/8 of the whole.

3 *num*erator or *numberer* of the parts
8 *denom*inator or *namer* of the parts

Here's another important idea for you to understand: the *larger* the denominator, the *smaller* the value of the fraction.



Look at the circles above. Compare the size of the shaded sections. Can you see that 1/2 is the largest section and 1/8 is the smallest? What would 1/16 look like?

Now suppose we took *two* of each section.

2/2, 2/3, 2/4, 2/5, 2/6, 2/7, 2/8

Which would be larger, 2/3 or 2/8? 2/4 or 2/6?

Job Assignment 1

Arrange these fractions in order of size, from the *smallest* to the *largest*.

Example: 1/4 1/2 1/6 1/6 1/4 1/2

1) 1/8 1/6 1/3

2) 1/5 1/16 1/32

3) 1/10 1/3 1/4

4) 2/5 2/8 2/3

5) 3/4 3/5 3/6

6)	1/4	1/16	1/8
7)	3/10	3/8	3/5
8)	4/6	4/10	4/32
9)	2/3	2/5	2/10
10)	3/16	3/12	3/3

There are three types of fractions we will use in this booklet. They are:

1. **Proper fractions** – the numerator is smaller than the denomoninator. Example: $\frac{6}{10}$ is a proper fraction.

2. **Improper fraction** – the numerator is greater than or equal to the denominator. Examples: 3/2 and 8/8 are improper fractions.

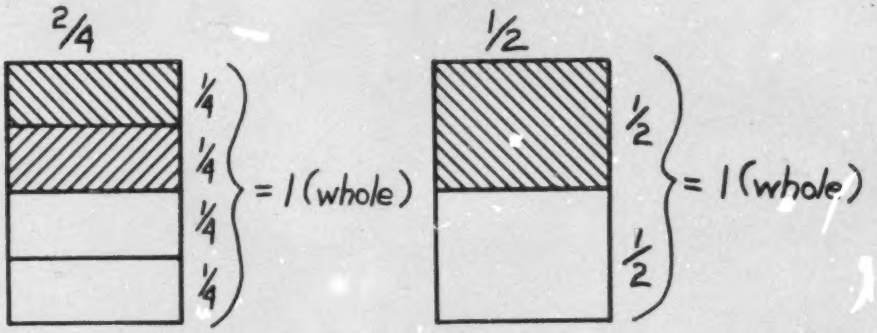
3. **Mixed numbers** – a whole number plus a fraction. Example: $3\frac{1}{3}$ is a mixed number. This means $3 + \frac{1}{3}$

Know the above definitions, because these words will be used throughout this booklet.

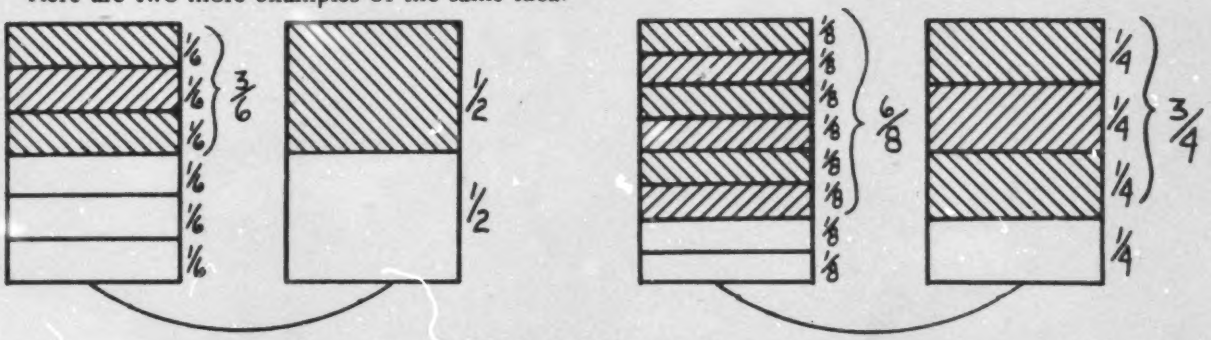
II – REDUCING AND CHANGING FRACTIONS

When you "reduce" a fraction, what you are doing is giving *another name* to the *same fraction*. Sometimes we use the word "simplify" to mean the same thing.

The following drawings will show that the fraction 2/4 (on the left) can be renamed 1/2 (as on the right) without changing its meaning.



Here are two more examples of the same idea.



Same fraction but different name

We said above that reducing gives us another name for the *same fraction*.

We might more accurately call them *equivalent fractions*. "Equivalent" means *equal in value*, and that is exactly what reduced fractions are – equal in value to the original fractions.

A reduced fraction makes solving problems easier. Also, many tools, like wrenches, are listed by their names as reduced fractions – 1/2", 1/4", 3/4", 3/8", 1/8", rather than 8/16", 4/16", 12/16", 6/16", 2/16", respectively.

To reduce a fraction:

1. Find a number that will divide evenly into both the numerator and the denominator.

2. Divide both parts by this number.

Example A: Reduce 6/8

Step I $\frac{6}{8}$ — Write the problem.

Step II $\frac{6 \div ?}{8 \div ?}$ — Think to yourself, "What number can divide evenly into both 6 and 8?"

Step III $\frac{6 \div 2}{8 \div 2}$ — The number 2 will divide evenly into both 6 and 8.

Step IV $\frac{6 \div 2}{8 \div 2} = \frac{3}{4}$ — 2 divides into 6, 3 times $2\overline{)6}^{3}$

2 divides into 8 , 4 times $2\overline{)8}^{4}$

Step V $\frac{3}{4}$ — This is the result of the dividing. See whether any other number will divide evenly into both parts. In this case there is none. Therefore 3/4 is another name for 6/8.

Example B: Reduce 9/12

Step I $\frac{9}{12}$ — Write the problem.

Step II $\frac{9 \div ?}{12 \div ?}$ — Think to yourself "What number will divide evenly into both 9 and 12?"

Step III $\frac{9 \div 2}{12 \div 2}$ — The number 3 will divide evenly into both 9 and 12.

Step IV $\frac{3}{4}$ — 3 divided into 9 = 3, numerator
3 divided in 12 = 4, denominator.
There is no other number that will divide evenly into both parts.

Example C: Reduce $\frac{20}{100}$

Step I	$\frac{20}{100}$	Write the problem
Step II	$\frac{20 \div ?}{100 \div ?}$	Think to yourself, "What number will divide evenly into both 20 and 100?"
Step III	$\frac{20 \div 5}{100 \div 5}$	5 will divide evenly into both 20 and 100.
Step IV	$\frac{4}{20}$	5 divided into 20 = 4, numerator 5 divided into 100 = 20, denominator
Step V	$\frac{4 \div ?}{20 \div ?}$	Is there any number that will divide evenly into both 4 and 20?
Step VI	$\frac{4 \div 4}{20 \div 4}$	4 will divide evenly into both 4 and 20.
Step VII	$\frac{1}{5}$	The reduced fraction. Some math whiz might have noticed at the beginning that 10 or even 20 would go evenly into both 20 and 100. In that case, the reducing could have been done more quickly.

We have now learned how to reduce or rename a fraction. Of course there are some fractions that cannot be reduced at all. For example, 3/8, or 5/6. There is no one number that will divide evenly into both numerator and denominator. Therefore they cannot be reduced.

Job Assignment 2

Reduce the following fractions wherever possible.

(A)

1) $\frac{5}{20}$
2) $\frac{4}{16}$
3) $\frac{6}{12}$
4) $\frac{7}{8}$
5) $\frac{2}{16}$
6) $\frac{5}{40}$
7) $\frac{3}{15}$
8) $\frac{100}{500}$
9) $\frac{7}{56}$
10) $\frac{4}{36}$
11) $\frac{7}{42}$
12) $\frac{8}{64}$
13) $\frac{1}{4}$
14) $\frac{4}{24}$
15) $\frac{10}{100}$
16) $\frac{9}{81}$
17) $\frac{2}{8}$
18) $\frac{13}{26}$

19) $\frac{11}{99}$

20) $\frac{14}{56}$

21) $\frac{7}{21}$

22) $\frac{5}{125}$

23) $\frac{8}{9}$

24) $\frac{17}{170}$

25) $\frac{25}{125}$

(B) 1) $\frac{2}{10}$

2) $\frac{16}{32}$

3) $\frac{8}{12}$

4) $\frac{9}{15}$

5) $\frac{25}{30}$

6) $\frac{7}{8}$

7) $\frac{18}{20}$

8) $\frac{10}{15}$

9) $\frac{15}{25}$

10) $\frac{24}{32}$

11) $\frac{3}{9}$

12) $\frac{4}{10}$

13) $\frac{9}{24}$

14) $\frac{16}{40}$

15) $\frac{8}{12}$

16) $\frac{10}{25}$

17) $\frac{25}{40}$

18) $\frac{18}{36}$

19) $\frac{14}{16}$

20) $\frac{28}{42}$

21) $\frac{125}{150}$

22) $\frac{200}{250}$

23) $\frac{10}{12}$

24) $\frac{75}{100}$

25) $\frac{3}{20}$

Mixed Numbers

You will remember that we defined a mixed number as a whole number plus a fraction. To reduce a mixed number, just reduce the fraction part – not the whole number.

Here's how to reduce a mixed fraction:

Example: Reduce $5\frac{9}{27}$

Step I	$5\frac{9}{27}$	Write the problem.
Step II	$5 + \frac{9}{27}$	Definition of a mixed number.
Step III	$5 + \frac{9 \div 9}{27 \div 9}$	Find a number that divides evenly into 9 and 27. 9 is the number.
Step IV	$5 + \frac{1}{3}$	$9 \div 9 = 1$. Write as numerator $27 \div 9 = 3$. Write as denominator
Step V	$5\frac{1}{3}$ Answer	Definition of a mixed number.

When reducing mixed numbers, it may be possible that the fraction part cannot be reduced. For example: $7\frac{3}{4}$ is a mixed number that cannot be reduced.

Job Assignment 3

Reduce the following mixed fractions where possible:

(1) $2\frac{3}{6}$

(2) $4\frac{6}{9}$

(3) $13\frac{12}{24}$

(4) $6\frac{12}{36}$

(5) $35\frac{9}{12}$

(6) $18\frac{2}{16}$

(7) $4\frac{4}{5}$

(8) $10\frac{6}{18}$

(9) $42\frac{30}{100}$

(10) $57\frac{11}{22}$

(11) $5\frac{8}{10}$

(12) $18\frac{6}{24}$

(13) $7\frac{9}{27}$

(14) $16\frac{7}{8}$

(15) $14\frac{10}{25}$

(16) $23\frac{8}{12}$

(17) $6\frac{12}{16}$

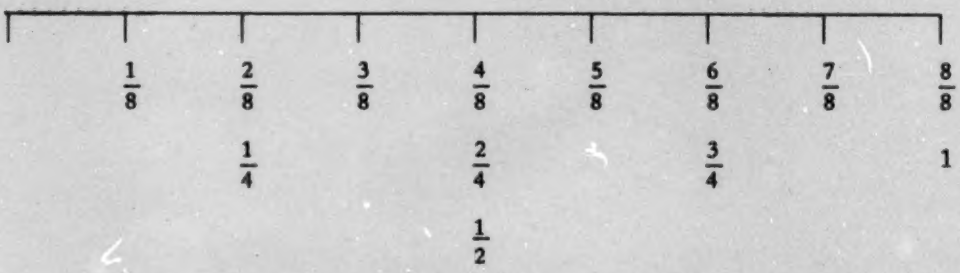
(18) $2\frac{5}{20}$

(19) $7\frac{16}{20}$

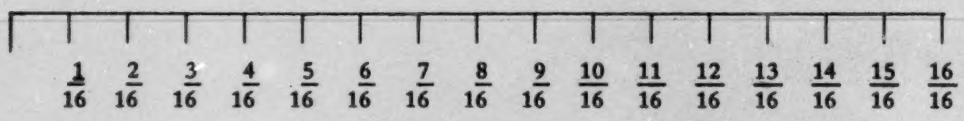
(20) $4\frac{21}{28}$

Job Assignment 4

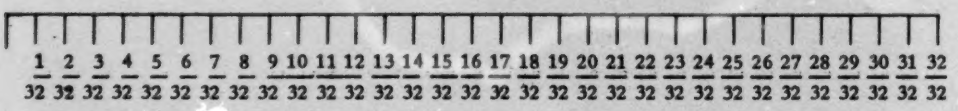
Below is a "picture" of an inch divided into eighths. Note that we have reduced all the eighths that could be reduced. The lowest figures in each case show the most common equivalent fractions.



(1) Using the above as a model, write the equivalent (reduced) fractions for an inch divided into 16ths:



(2) Do the same for an inch divided into 32nds.



Changing Improper Fractions to Mixed Numbers:

Suppose you were a cook and had to double a recipe calling for 2/3 cup of sugar. You got 4/3 of a cup. Of course, you wouldn't proceed by measuring 1/3 of a cup 4 times. You'd want to know the measurement in terms of *whole cups.*

Or suppose you were a carpenter and had to rout out a single space for two 3/4-inch boards. You knew you had to allow 6/4 inches. How many inches would you have to rout out?

In both these cases we have the problem of simplifying an improper fraction, that is, a fraction with the numerator larger than the denominator. We need to change these fractions to *equivalent mixed numbers.*

To change an improper fraction to a whole or mixed number:

1. Divide the denominator into the numerator.
2. Put any remainder from the division over the denominator.
3. Reduce the remaining fraction if it can be reduced.

Example A: Change 19/8 to a whole or mixed number.

Step I	$\frac{19}{8}$	Write the problem.
Step II	$8\overline{)19}$	Divide the denominator into the numerator.
Step III	$8\overline{)\begin{array}{r}2\text{ R3}\\19\end{array}}$	8 goes into 19, 2 times, with a remainder of 3.
Step IV	$\frac{19}{8} = 2\frac{3}{8}$	Answer, with the remainder expressed as a fraction.

Example B: Change 16/4 to a whole or mixed number.

Step I	$\frac{16}{4}$	Write the problem.
Step II	$4\overline{)16}$	Divide the denominator into the numerator.
Step III	$4\overline{)\begin{array}{r}4\\16\end{array}}$	4 divides into 16, 4 times, with no remainder.
Step IV	$\frac{16}{4} = 4$	The answer is the whole number 4.

Example C: Change 9/6 to a whole or mixed number.

Step I	$\frac{9}{6}$	Write the problem.
Step II	$6\overline{)9}$	Divide 6 into 9 (the denominator into the numerator)
Step III	$1\frac{3}{6}$	6 divides into 9, 1 time, with a remainder of 3. Express the remainder as a fraction.
Step IV	$1\frac{3}{6} = 1\frac{3 \div 3}{6 \div 3} = 1\frac{1}{2}$	Reduce the fraction by dividing numerator and denominator by 3.

Job Assignment 5

Change these improper fractions to whole numbers or mixed numbers. Reduce fractions wherever possible.

(1)	$\frac{4}{3}$	(11)	$\frac{45}{3}$
(2)	$\frac{6}{4}$	(12)	$\frac{18}{12}$
(3)	$\frac{11}{2}$	(13)	$\frac{24}{4}$
(4)	$\frac{17}{10}$	(14)	$\frac{13}{10}$
(5)	$\frac{9}{4}$	(15)	$\frac{14}{3}$
(6)	$\frac{23}{8}$	(16)	$\frac{7}{2}$
(7)	$\frac{20}{5}$	(17)	$\frac{20}{8}$
(8)	$\frac{65}{10}$	(18)	$\frac{35}{6}$
(9)	$\frac{14}{8}$	(19)	$\frac{34}{16}$
(10)	$\frac{11}{3}$	(20)	$\frac{250}{100}$

Changing Mixed Numbers to Improper Fractions

To this point we have learned how to reduce fractions and how to change improper fractions to whole or mixed numbers. Now we must learn one more skill before we can go on. Suppose we needed to take twice a certain measurement, and the measurement turned out to be 2¾ inches. In order to do this easily, we would need to change the 2¾ to an improper fraction.

To change a mixed number to an improper fraction:

1. Multiply the whole number by the denominator of the fraction.
2. Add the numerator of the fraction to that product.
3. Place this final amount as the numerator over the old denominator.

Example A: Change 5½ to an improper fraction:

Step I	$5\frac{1}{2}$	Write the problem.
Step II	$5 \times 2 + 1 = 11$	Multiply the whole number 5 by the denominator 2; then add the numerator 1 to the product.
Step III	$5\frac{1}{2} = \frac{11}{2}$	Write the total from Step II over the old denominator for the answer.

Job Assignment 6

A. Draw a row of 5 squares and divide each in half. At the end of the row draw a half of a square. This will give you 5½ squares. Count up the number of half-squares. How many do you get? Compare Step III above.

B. Now divide each half into half again. Express what you have as
(1) a mixed number
(2) an improper fraction

Example C: Change the following mixed numbers to improper fractions.

(1) $4\frac{1}{2}$

(2) $3\frac{1}{3}$

(3) $10\frac{1}{4}$

(4) $7\frac{2}{3}$

(5) $12\frac{1}{2}$

(6) $6\frac{7}{8}$

(7) $9\frac{4}{5}$

(8) $10\frac{3}{8}$

(9) $25\frac{1}{4}$

(10) $8\frac{5}{6}$

(11) $12\frac{3}{4}$

(12) $9\frac{3}{10}$

(13) $7\frac{2}{3}$

(14) $13\frac{1}{5}$

(15) $4\frac{5}{8}$

(16) $2\frac{9}{16}$

(17) $4\frac{1}{6}$

(18) $2\frac{7}{10}$

(19) $100\frac{4}{5}$

(20) $7\frac{3}{8}$

(21) Change 5 to an improper fraction with a denominator of 4.

(22) Change 6 to an improper fraction with a denominator of 3.

III – MULTIPLYING FRACTIONS

A carpenter needs 6 boards for the shelves of a cabinet, each 2¼ feet long. How many feet of shelving does he need? Clearly, he has to multiply 2¼ by 6 in order to find out.

In this section we shall learn to multiply fractions. Let's first consider a proper fraction multiplied by another, as 1/2 multiplied by 2/3.

To multiply fractions:

1. Write the problem with a "times" sign written between the fractions.
2. Multiply the numerators together, then multiply the denominator.
3. Reduce the resulting fraction, if possible.

Example A: Multiply 1/2 by 2/3.

Step I	$\frac{1}{2} \times \frac{2}{3}$	Write the problem.
Step II	$\frac{1 \times 2}{2 \times 3} = \frac{2}{6}$	Multiply the numerators together. Multiply the denominators together.
Step III	$\frac{2}{6} = \frac{1}{3}$ Answer	2/6 can be reduced to 1/3.

Very often we need to multiply a fraction by a whole number. Any whole number can be written as a fraction simply by placing it over a denominator of 1. (This divides it by 1, which does not change its value in any way.)

Example B: Multiply 3/4 by 7.

Step I	$\frac{3}{4} \times \frac{7}{1}$	Write the problem, expressing the whole number 7 as a fraction with a denominator of 1.
Step II	$\frac{3}{4} \times \frac{7}{1} = \frac{3 \times 7}{4 \times 1} = \frac{21}{4}$	Multiply numerators together and denominators together
Step III	$\frac{21}{4}$ = Answer	21/4 is an improper fraction. To change an improper fraction to a mixed number, divide the denominator into the numerator.

Job Assignment 7

Multiply the following fractions. Reduce answer where possible, and change any improper fractions to mixed numbers.

(1) $\frac{1}{3} \times \frac{3}{4} =$

(2) $\frac{1}{2} \times \frac{4}{5} =$

(3) $\frac{1}{2} \times \frac{1}{2} =$

(4) $\frac{3}{8} \times \frac{3}{8} =$

(5) $\frac{1}{2} \times \frac{13}{16} =$

(6) $\frac{3}{4} \times \frac{3}{4} =$

(7) $2 \times \frac{3}{4} =$

(8) $\frac{1}{5} \times 5 =$

(9) $12 \times \frac{1}{3} =$

(10) $3 \times \frac{5}{6} =$

(11) $\frac{3}{4} \times 6 =$

(12) $\frac{3}{4} \times 16 =$

(13) $\frac{1}{2} \times 240 =$

(14) $8 \times \frac{2}{3} =$

(15) $9 \times \frac{2}{5} =$

(16) $13 \times \frac{1}{4} =$

(17) $\frac{3}{8} \times 8 =$

(18) $\frac{5}{8} \times 24 =$

(19) $15 \times \frac{2}{3} =$

(20) $15 \times \frac{3}{4} =$

Multiplying by a Mixed Number

Our carpenter above had to multiply 2¼ by 6. He could have multiplied 2 by 6, and 1/4 by 6, and added the results together. But having studied fractions, he did it the easy way. He changed the 2¼ to an improper fraction, and then it was a simple matter to multiply by 6.

We will take it step by step.

Example C: Multiply 2¼ by 6.

Step I $2\frac{1}{4} \times 6$ Write the problem.

Step II $\frac{9}{4} \times \frac{6}{1}$ Express both numbers as improper fractions.

Step III	$\frac{9}{4} \times \frac{6}{1} = \frac{54}{4}$	Multiply numerators together. Multiply denominators together.
Step IV	$\frac{54}{4} = \frac{27}{2}$	Reduce the fraction (before or after simplifying – it is easier before).
Step V	$\frac{27}{2} = 13\frac{1}{2}$	Change to a mixed number.

Summary – Multiplication Involving Fractions

Step I	Write the problem.
Step II	If you have a mixed number to multiply, first change it to an improper fraction. If you have a whole number, change it to an improper fraction with a denominator of 1.
Step III	Multiply numerators together and denominators together. (You may also divide numerator and denominator by the same number where possible. ("Canceling" was the old term for this.)
Step IV	Reduce the fraction whenever possible. Change an improper fraction to a mixed number.

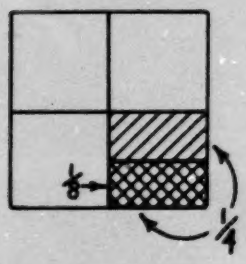
There is something unusual about multiplying by fractions, which you may not have noticed. If you multiply *any number* by a *proper fraction*, you end up with a *smaller* number than you started out with.

Actually this is logical. If you take ½ of something, or 2/3 of something, or ¾ of something – you are going to get less than the thing itself.

$$16 \times \frac{3}{4} = \frac{16}{1} \times \frac{3}{4} = \frac{48}{4} = 12$$

and 12 is less than 16.

If you multiply a proper fraction by a proper fraction, you will get a *very small* fraction. For example, one-half of one-fourth equals one-eighth of the whole.



Here is a picture of 1/2 of 1/4 of a square. Note that you get 1/8 of the original square.

Job Assignment 8

(A) Go back to job assignment 7, problems 7 through 20, and see whether each answer *is* actually smaller than the whole number in the problem.

(B) In problems 1–6 of the same job assignment, consider the second fraction as the multiplier of the first and check to see whether each answer is actually smaller than the first fraction.

(C) Multiply the following mixed numbers. Reduce all answer fractions and change improper fractions to mixed numbers.

(1) $1\frac{2}{3} \times 3$

(2) $3\frac{3}{4} \times 2$

(3) $4 \times 4\frac{1}{2}$

(4) $6 \times 3\frac{1}{2}$

(5) $1\frac{2}{3} \times 7$

(6) $4 \times 3\frac{1}{4}$

(7) $1\frac{4}{5} \times 5$

(8) $2\frac{1}{2} \times 2\frac{1}{2}$

(9) $4 \times 3\frac{1}{3}$

(10) $\frac{1}{2} \times 7\frac{1}{2}$

(11) $6\frac{3}{4} \times \frac{1}{3}$

(12) $5 \times 2\frac{1}{8}$

(13) 7×5

(14) $\frac{3}{4} \times 2\frac{1}{3}$

(15) $1\frac{1}{2} \times 1\frac{1}{3}$

(16) $\frac{1}{6} \times 4\frac{2}{3}$

(17) $2\frac{1}{4} \times 2\frac{1}{2}$

(18) $\frac{1}{5} \times \frac{1}{10}$

(19) $\frac{1}{5} \times 12\frac{1}{2}$

(20) $\frac{5}{6} \times 9$

(21) $\frac{1}{2} \times \frac{2}{3} \times 3$

(22) $\frac{3}{4} \times \frac{2}{3} \times 6$

(23) $\frac{2}{3} \times 7$

(24) $1\frac{1}{2} \times 2\frac{1}{2}$

(25) $3\frac{1}{3} \times 10$

Job Assignment 9

The Independent Manufacturing Company permits its workers to set up their own schedules of work hours. Below is a list of some of its workers, the hours they work per day, and the number of days they worked during one week. Figure out the total number of hours each one worked, and his or her gross pay for the week.

Worker	Hours/ Day	Days Worked	Total Hours	Wage Per Hour	Gross Pay
Brandt, Joan	6	5½		$3.00	
Collins, Bertha	7½	4		$6.00	
Filardi, Steve	5¾	5		$5.00	
Evans, Joe	8	6		$5.00	
Harris, Celeste	5½	4		$8.00	
Lukas, Henry	3¾	6		$10.00	
Rizek, Phil	8	5½		$5.00	
Williams, Jeff	6¼	5		$6.00	

Job Assignment 10

In carpentry, lumber is measured in board-measure, often called board-feet. The unit is a square foot 1 inch thick. To find how many board-feet are needed, multiply the thickness of the lumber in inches by the width (in feet) by the length of the lumber (in feet). Find the number of board-feet needed for each size of lumber listed below. (Note that the width is given in inches and you need the width in feet. Convert inches to feet by dividing by 12. Reduce the fraction to make the figuring easier.)

Quantity	Thickness	Width	Length	No. of Board-feet
12	1"	6"	8'	
24	1"	6"	12'	
14	4"	10"	16'	
60	2"	4"	8'	
16	2"	4"	12'	
16	2"	8"	14'	
20	6"	6"	4'	
18	4"	4"	10'	
12	4"	4"	8'	
10	1"	10"	10'	

Job Assignment 11

Reduce all answers to lowest terms.

1. Ripping a 3¾-inch board in half gives two boards of what width?

2. If one washer is $\frac{3}{16}$ of an inch thick, what is the thickness of 2 washers?

3. The distance between the centers of rivets is 2 5/16 inches. What is the distance center-to-center between the first and eighth rivets?

4. A recipe calling for 1 2/3 cups of flour is to be tripled (multiplied by 3). How much flour will be needed?

5. A beginning machinist was producing threaded bolts. Half the bolts were produced with a dull cutting tool, and ¾ of these were rejected. What fraction of the total amount produced was rejected?

6. Leon had a job mowing the neighbor's lawn each week. It took him 1¼ hours each time. How many hours did he actually work in 12 weeks?

7. Three girls in the Power Machine Shop at school had a total of 42 collars to sew. (a) Express each girl's share of the work as a fraction. (b) How many collars did each sew?

8. A drapery maker had to make 3 pairs of drapes (6 pieces in all), for each of which she needed 2 7/8 yards. How much fabric did she have to order?

9. How high up on a wall will 16 rows of tile come if each row is 4¼ inches tall? (Express answer in feet).

10. If a certain antenna wire runs 18 feet to the pound, how many feet can the installer figure on if he had a coil left that weighs 1¾ pounds?

IV – DIVIDING FRACTIONS

A builder is planning a flight of stairs whose total height is to be 8½ feet. If each step is to be 8 inches high, how many steps will be required? This is clearly a problem in division, but what does the builder divide by what? Knowing his fractions, the builder changes the 8 inches to feet:

$$\frac{8}{12} = \frac{2}{3} \text{ foot}$$

He then proceeds to divide 8½ feet by 2/3 of a foot. Let us see how he does it.

To divide fractions:

1. *Invert* the divisor – turn it "upside down."
2. Multiply the two numbers.

The *divisor,* you will remember, is the number you are *dividing by*. In the above problem you are dividing by 2/3 foot, so that is the number you invert.

Example A: Divide 8½′ by 2/3′

Step I	$8\frac{1}{2} \div \frac{2}{3}$	Write the problem.
Step II	$\frac{17}{2} \div \frac{2}{3}$	Change the mixed number to an improper fraction.
Step III	$\frac{17}{2} \times \frac{3}{2}$	Invert the divisor and change ÷ to X.
Step IV	$\frac{17 \times 3}{2 \times 2} = \frac{51}{4}$	Multiply the fractions.
Step V	$\frac{51}{4} = 12\frac{3}{4}$	Change answer to a mixed number.

(Our builder cannot have 12¾ steps, so he must figure on 13 steps and make a small adjustment in height.)

One problem that comes up frequently is that of dividing a mixed number by a whole number. A typist is making a chart 6½ inches wide, and she needs 4 equal columns. How wide must each column be?

Before we see how to do this problem, recall that any whole number can be expressed as an improper fraction by putting it over a denominator of 1.

Example B: Divide 6½" by 4.

Step I	$6\frac{1}{2} \div 4$	Write the problem.
Step II	$\frac{13}{2} \div \frac{4}{1}$	Express both terms as improper fractions.
Step III	$\frac{13}{2} \div \frac{1}{4}$	Invert the divisor and change ÷ to X.
Step IV	$\frac{13 \times 1}{2 \times 4} = \frac{13}{8} = 1\frac{5}{8}$"	Multiply across and simplify answer.

If *dividing* by inverting the divisor and *multiplying* does not seem to make sense to you, consider this problem:

A sheet-metal worker has a piece of metal 4 feet wide and needs to cut it into strips that are 6 inches (½ foot) wide. How many strips will he be able to get? Divide 4 feet by ½ foot.

$$4 \div \frac{1}{2} = \frac{4}{1} \div \frac{1}{2} = \frac{4}{1} \times \frac{2}{1} = 8 \text{ strips}$$

Now let's look at those strips another way – since each is ½ foot wide, there will be 2 strips to each foot. Well, with 2 strips to *each* foot, how many strips will there be in 4 feet? *Multiply* 4 feet by 2 strips per foot.

$$4 \times 2 = 8 \text{ strips}$$

So you see that dividing by ½ *is* really the same as multiplying by 2.

Let's look at another problem. If your bakeshop had baked 800 cupcakes for the school picnic and wanted each class to have an equal number, how would you find the number? You would *divide* by 4. Let's call that Method A.

You could look at the problem a little differently and say that each class would get *one-quarter* of the total. Then you would take ¼ of 800, which is a problem in *multiplication*. Let's call that Method B.

Method A: $800 \div 4 = \frac{800}{1} \div \frac{4}{1} = \frac{800}{1} \times \frac{1}{4}$ (which is Method B)
Whichever method you use, you end up with 200 cupcakes per class.

One more point – notice that, when you divide any number by a *proper fraction*, you end up with a *larger* number than you started out with.

For example, $16 \div \frac{1}{2} = \frac{16}{1} \div \frac{1}{2} = \frac{16}{1} \times \frac{2}{1} = 32$

Job Assignment 12

(A) Tell whether your answer to each of the following will be larger or smaller than the original number.

(1) 8½ ÷ 2/3 Answer will be _____ than 8½.

(2) 6½ ÷ 4 Answer will be _____ than 6½.

(3) 10 ÷ ½ Answer will be _____ than 10.

(4) 3½ ÷ 2 Answer will be _____ than 3½.

(5) 10 ÷ ¼ Answer will be _____ than 10.

(B) Do the division problems in (A) above.

(C) Divide the following. Give all answers in fractions or mixed numbers in lowest terms.

(1) $\frac{3}{4} \div \frac{1}{2}$

(2) $\frac{3}{4} \div 2$

(3) $3\frac{1}{2} \div 5$

(4) $2\frac{7}{8} \div \frac{3}{4}$

(5) $72 \div 1\frac{1}{2}$

(6) $3\frac{1}{2} \div 12$

(7) $9\frac{3}{4} \div 13$

(8) $48 \div 5\frac{1}{2}$

(9) $\frac{2}{3} \div \frac{2}{3}$

(10) $\frac{1}{2} \div \frac{1}{2}$

(11) $\frac{2}{5} \div 1\frac{1}{2}$

(12) $96 \div 6$

(13) $\frac{15}{16} \div 3$

(14) $12 \div \frac{1}{2}$

(15) $12\frac{1}{2} \div 3$

(16) $3\frac{1}{4} \div 4$

(17) $54 \div 3\frac{1}{3}$

(18) $15 \div \frac{1}{2}$

(19) $\frac{5}{8} \div 4$

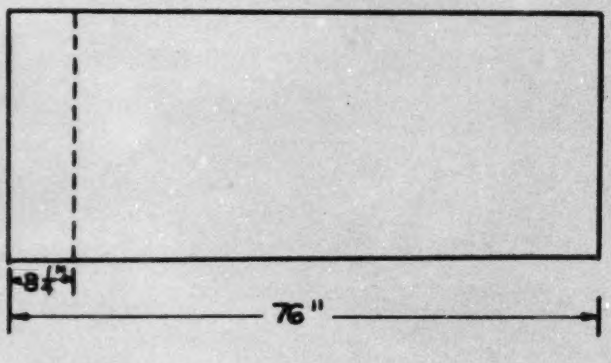
(20) $\frac{7}{16} \div \frac{7}{12}$

Job Assignment 13

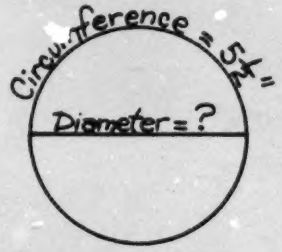
Solve the following:

1. How many $2\frac{1}{4}'$ pieces of wood can be cut from a board 12' long?

2. A board is 76" long. From this board we wish to cut pieces $8\frac{1}{4}$" wide. How many full pieces can be cut? How much waste wood is left? (This may be an approximation)



3. Four holes are to be evenly spaced across this block of steel 6¾" wide, allowing 1" from each end to the center of the nearest hole. What will the center-to-center distances be?



4. A man had some 5½-foot lengths of wire and wanted to form them into circles to make plant supports out of them. What would be the diameter of the circles he could make? Diameter = $\frac{\text{circumference}}{3\frac{1}{7}}$

5. How many ½-cup servings of applesauce can be made out of 1 gallon of applesauce? (1 gal. = 16 cups)

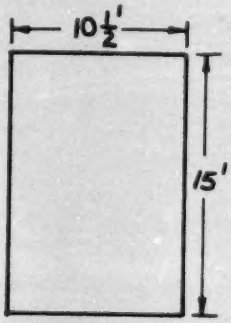
6. How many rows 1½ feet apart can be laid out in a portion of a garden that is 15½ feet wide?

7. If a course (row) of brick with its mortar is 3 inches high, how many courses will be needed for a wall 4½ feet high?

8. A stoop is 3½ feet above the sidewalk. How many steps will be needed if each step is to be 8 inches high?

9. A teaspoon is one-third of a tablespoon. How many teaspoons are there in 12 tablespoons?

10. How many 9" x 9" tiles can be laid on the floor illustrated here?



Quiz on Multiplying and Dividing Fractions

Perform the indicated operations. Reduce all answers.

Multiply

(1) $\frac{3}{8} \times 5$

(2) $\frac{1}{2} \times 2\frac{3}{5}$

(3) $12 \times 5\frac{1}{4}$

(4) $3\frac{2}{3} \times 6$

(5) $6\frac{2}{3} \times 2\frac{1}{5}$

(6) $8\frac{2}{5} \times 6$

(7) $14 \times 1\frac{1}{2}$

(8) $3\frac{1}{3} \times 15$

Divide

(9) $8 \div \frac{4}{5}$

(10) $4\frac{1}{5} \div 3$

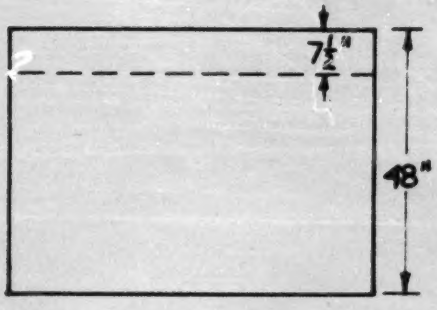
(11) $6\frac{1}{4} \div 5$

(12) $3\frac{2}{3} \div 4$

(13) $\frac{5}{8} \div \frac{1}{4}$

(14) Find the number of board feet needed, given the following information:

QUANTITY	THICKNESS	WIDTH	LENGTH
16 Boards	2"	8"	10'



How many pieces of finished lumber 7½" wide are needed to make the top of this table?

V – FINDING A COMMON DENOMINATOR

In making measurements and working with them, we often have to add or subtract fractions. Many times it is very simple and direct: adding 1/8 to 3/8 is as easy as adding 1 orange to 3 oranges. Instead of getting the sum of 4 oranges, we get the sum of 4/8 (which we generally reduce to 1/2).

So long as the denominators are alike, we can add fractions as easily as whole numbers. We add the numerators and put the result over the denominator.

But sometimes we have to add fractions with different denominators: 2/3 + 1/2, or 3/4 + 1/3. In order to do this, we must first learn to find a *common* denominator. ("Common" in this sense means "common" to both – in other words, the same for both.)

Once we have found the common denominator, we can proceed to change the fractions to equivalent fractions having the common denominator. Only then can we add or subtract them.

Let us look at a couple of examples, which will make clear how we would go about this.

Suppose we had to add

$$\frac{1}{2} + \frac{3}{4} + \frac{5}{8}$$

The three denominators are 2, 4, and 8. If we are to get equivalent fractions with the same denominator, we must find *one number that all of the denominators will divide into evenly* (with no remainder). Let us choose the largest denominator – 8. Then we will test the other denominators against the 8. Does 2 go into 8 evenly? Yes, it does. Does 4 go into 8 evenly? Yes, it does also. Obviously 8 also goes into 8 evenly. Therefore 8 is the common denominator we are looking for.

This problem was not too difficult, but other problems may be more difficult. You may remember from your early grades that there was a complicated system for finding common denominators. This booklet, however, deals only with simple fractions that you are likely to meet in your daily life – halves, quarters, eighths; thirds on occasion; sometimes fifths or tenths. Accurate measurements sometimes involve 16ths or even 32nds of an inch, and you may find a problem or two using them.

We will be able to find common denominators for these numbers just by carefully looking at them and thinking. For example, find a common denominator for

$$\frac{1}{4}, \frac{1}{6}, \text{ and } \frac{3}{8}$$

Again, we will start with the largest denominator – 8. We will test the other denominators to see whether they divide evenly into 8. The 4 does, but the 6 does not. Therefore 8 cannot be a common denominator for these three fractions. What to do? Still using the largest denominator, we will multiply it (in our heads) by small numbers *until we find a number that all the other denominators divide into evenly.*

8 x *2* = 16. Does 4 go evenly into 16? *Yes* (4 times). Does 6? *No.*

8 x *3* = 24. Does 4 go evenly into 24? *Yes* (6 times). Does 6? *Yes* (4 times).

Good! Now we have our common denominator – 24. We know that 8 goes evenly into 24, because that's how we got the number in the first place – by multiplying 8 by 3. We have also tested the other denominators and have found that they too go into 24.

Furthermore, by working this way, we have found the *smallest* common denominator, which will make our problem-solving easier. The smallest, or *least common denominator,* you may remember, is your old friend (or enemy) from elementary school, often called LCD.

Let's look together at one more example, before you go on your own. Find a common denominator for

$$\frac{1}{3} \text{ and } \frac{1}{4}$$

The larger denominator is 4. Will 3 go into it? No. Therefore 4 is not the LCD. Multiply 4 by *2*. 4 x 2 = 8. Will 3 go into 8 evenly? *No.* Therefore 8 is not the LCD. Try again – multiply 4 by *3*: 4 x 3 = 12. Will 3 go into 12 evenly? *Yes.* We have found the LCD – 12.

We will summarize the procedure for finding the least common denominator:

1. Select the largest denominator.
2. If all the other denominators can divide into it evenly, then that number is the LCD.
3. If one or more of the other denominators does not divide into it evenly, then multiply the largest denominator by 2. Test the other denominators.
4. If all the other denominators can divide into that number evenly, then that is the LCD.
5. If one or more of the other denominators does not divide into that number evenly, then multiply the largest denominator by 3.
6. Continue testing and multiplying the largest denominator by small numbers until you get a number that every denominator will divide into evenly. That number is the LCD.

Job Assignment 14

Find the LCD for each of the following groups.

(1) $\frac{2}{3}\ \frac{1}{6}\ \frac{5}{12}$

(2) $\frac{7}{8}\ \frac{1}{2}\ \frac{3}{4}$

(3) $\frac{1}{2}\ \frac{3}{4}\ \frac{5}{16}$

(4) $\frac{1}{2}\ \frac{1}{3}\ \frac{1}{4}$

(5) $\frac{1}{2}\ \frac{1}{4}\ \frac{1}{8}$

(6) $\frac{5}{8}\ \frac{7}{16}\ \frac{11}{32}$

(7) $\frac{3}{4}\ \frac{5}{6}\ \frac{1}{12}$

(8) $\frac{3}{5}\ \frac{1}{5}\ \frac{7}{10}$

(9) $\frac{3}{5}\ \frac{1}{3}\ \frac{4}{5}$

(10) $\frac{5}{8}\ \frac{7}{8}\ \frac{13}{16}$

(11) $\frac{1}{6}\ \frac{2}{3}\ \frac{5}{6}$

(12) $\frac{3}{4}\ \frac{1}{2}\ \frac{7}{32}$

(13) $\frac{2}{3}\ \frac{3}{4}\ \frac{1}{12}$

(14) $\frac{13}{16}\ \frac{1}{2}\ \frac{3}{4}$

(15) $\frac{5}{6}\ \frac{1}{3}\ \frac{3}{8}$

(16) $\frac{1}{3}\ \frac{1}{5}\ \frac{1}{10}$

(17) $\frac{2}{3}\ \frac{2}{5}\ \frac{1}{3}$

(18) $\frac{3}{4}\ \frac{1}{8}\ \frac{1}{4}$

(19) $\frac{1}{4}\ \frac{2}{5}\ \frac{7}{10}$

(20) $\frac{1}{10}\ \frac{7}{20}\ \frac{13}{30}$

Equivalent Fractions in Higher Terms

You know how to reduce fractions to lower terms. Now you will learn how to raise fractions to *higher* terms. (This will enable you to add fractions having different denominators.)

We *reduced* fractions by *dividing* both numerator and denominator by the same number.

$$\frac{4}{6} = \frac{4 \div 2}{6 \div 2} = \frac{2}{3}$$

Now we will do the opposite; we will *multiply* both numerator and denominator by the same number. As before, this will not change the value of the fraction.

$$\frac{2}{3} = \frac{2 \times 2}{3 \times 2} = \frac{4}{6}$$

Also, $\frac{2}{3} = \frac{2 \times 5}{3 \times 5} = \frac{10}{15}$

Also, $\frac{2}{3} = \frac{2 \times 10}{3 \times 10} = \frac{20}{30}$

This could go on forever! You can raise a fraction to higher terms by multiplying numerator and denominator by any number at all.

The question is, what number to use to multiply by?

Suppose we needed to change $\frac{2}{3}$ to an equivalent fraction with a *denominator of 6:*

$$\frac{2}{3} = \frac{?}{6}$$

Since 6 is larger than 3, we know we must raise the fraction $\frac{2}{3}$ to higher terms.

To raise a fraction to higher terms:

1. Divide the denominator of the fraction into the new denominator.
2. Multiply both numerator and denominator by the quotient you obtain.

Example A: Change 2/3 to sixths.

Step I $\frac{2}{3} = \frac{?}{6}$ Write the problem.

Step II $\frac{2}{3} = \frac{\ }{6}$ Divide 6 by 3. Quotient is 2.

Step III $\frac{2 \times 2}{3 \times 2} = \frac{4}{6}$ Answer Multiply numerator and denominator by 2.

Example B: Change 3/4 to 12ths.

Step I $\frac{3}{4} = \frac{?}{12}$ Write the problem.

Step II $\frac{3}{4} = \frac{\ }{12}$ Divide 12 by 4. Quotient is 3.

Step III $\frac{3 \times 3}{4 \times 3} = \frac{9}{12}$ Answer Multiply numerator and denominator by 3.

Job Assignment 15

Raise these fractions to equivalent fractions having the denominators indicated.

(1) $\frac{2}{3} = \frac{?}{6}$

(2) $\frac{1}{4} = \frac{?}{12}$

(3) $\frac{1}{5} = \frac{?}{10}$

(4) $\frac{3}{4} = \frac{?}{8}$

(5) $\frac{1}{3} = \frac{?}{12}$

(6) $\frac{1}{4} = \frac{?}{16}$

(7) $\frac{3}{4} = \frac{?}{12}$

(8) $\frac{5}{6} = \frac{?}{12}$

(9) $\frac{2}{3} = \frac{?}{12}$

(10) $\frac{1}{4} = \frac{?}{20}$

(11) $\frac{3}{8} = \frac{?}{16}$

(12) $\frac{1}{3} = \frac{?}{30}$

(13) $\frac{3}{4} = \frac{?}{32}$

(14) $\frac{7}{8} = \frac{?}{24}$

(15) $\frac{2}{3} = \frac{?}{24}$

(16) $\frac{5}{6} = \frac{?}{12}$

(17) $\frac{1}{4} = \frac{?}{24}$

(18) $\frac{3}{5} = \frac{?}{30}$

(19) $\frac{2}{3} = \frac{?}{18}$

(20) $\frac{5}{8} = \frac{?}{16}$

Job Assignment 16

Go back to job assignment 14 and express each fraction in terms of the common denominator you found.

(1) $\frac{2}{3}$ $\frac{1}{6}$ $\frac{5}{12}$ $\frac{8}{12}$ $\frac{2}{12}$ $\frac{5}{12}$

(2)

(3)

(4)

(5)

(6)

(7)

(8)

(9)

(10)

(11)

(12)

(13)

(14)

(15)

(16)

(17)

(18)

(19)

(20)

VI – ADDING FRACTIONS

After all this preparation, we are finally ready to practice addition of fractions.

First, as we said on page 26, when the fractions have the *same denominator,* adding them is like adding oranges – or apples – or any other whole things.

To add fractions that have a common denominator:

1. Add the numerators together.
2. Place that sum over the original denominator.
3. If the result is a proper fraction, reduce it if possible.
4. If the result is an improper fraction, change it to a whole number or improper fraction (in lowest terms).

Example A: Add 9/16 and 3/16.

Step I $\frac{9}{16}$ + $\frac{3}{16}$

Write the problem. Note that the denominators are the same.

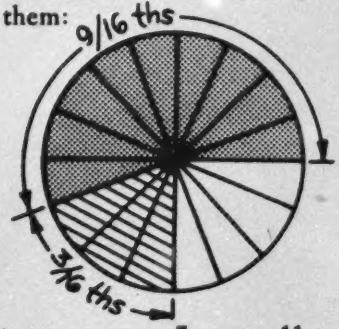
Step II $\frac{12}{16}$

Add the numerators only. Place the sum of the numerators over the denominators.

Step III $\frac{12 \div 4}{16 \div 4} = \frac{3}{4}$ Answer

In this case, the sum can be reduced by dividing both numerator and denominator by 4.

Note: Some students feel that, since they are adding numerators, they must also add denominators. (They forget about the oranges and apples!) Perhaps this will help them:



You can see that 9/16 + 3/16 really does equal 12/16, which really does equal 3/4 of the whole. (What part of the whole would 12/*32* be?)

Example B: Add $\frac{7}{12}$ and $\frac{11}{12}$

Step I $\frac{7}{12}$ + $\frac{11}{12}$

Write the problem.

Step II $\frac{18}{12}$

Since the denominators are the same, add the numerators and place the sum over the original denominator.

Step III $\frac{18}{12} = 1\frac{6}{12}$ Since the sum is an improper fraction, divide the denominator into the numerator to change it to a mixed number.

Step IV $1\frac{6}{12} = 1\frac{1}{2}$ Answer Reduce to lowest terms.

Job Assignment 17

Add the following fractions. Reduce all answers.

(1) $\frac{3}{8} + \frac{3}{8} =$ (Ans)

(2) $\frac{3}{4} + \frac{1}{4} =$

(3) $\frac{3}{8} + \frac{7}{8} =$

(4) $\frac{5}{12} + \frac{7}{12} =$

(5) $\frac{2}{5} + \frac{3}{5} + \frac{4}{5} =$

(6) $\frac{5}{6} + \frac{5}{6} + \frac{1}{6} =$

(7) $\frac{3}{8} + \frac{5}{8} + \frac{7}{8} =$

(8) $\frac{7}{12} + \frac{1}{12} + \frac{11}{12} =$

(9) $\frac{1}{4} + \frac{1}{4} + \frac{1}{4} =$

(10) $\frac{5}{8} + \frac{1}{8} + \frac{7}{8} + \frac{3}{8} =$

(11) $2\frac{1}{4} + 4\frac{1}{4}$

(12) $3\frac{1}{3} + 6\frac{1}{3}$

(13) $3\frac{3}{4} + 4$

(14) $2\frac{3}{8} + \frac{3}{8}$

(15) $10\frac{1}{2} + \frac{1}{2}$

To add fractions of different denominators:

1. Find a common denominator.
2. Change each fraction to an equivalent fraction having the common denominator.
3. Add numerators together.
4. If the sum is a proper fraction, reduce the fraction to its lowest terms.
5. If the sum is an improper fraction, change it to a whole or mixed number, in lowest terms.

Example A: Add 1/2 and 3/8.

Step I $\frac{1}{2}$
$+ \frac{3}{8}$

Write the problem. Note that the denominators are not the same.

Step II Common Denom. = 8

By testing 2 against 8, decide that 8 can be the common denominator.

Step III $\frac{1}{2} = \frac{4}{8}$
$\frac{3}{8} = \frac{3}{8}$

Change to equivalent fractions having the denominator 8.

Step IV $\frac{7}{8}$ Answer

Add, as before.

Example B: Add 3/4 and 2/3.

Step I $\frac{3}{4}$
$+ \frac{2}{3}$

Write the problem. Note that a common denominator will have to be found.

Step II Common Denom. = 12

By testing the denominators against each other, decide that 12 can be the common denominator.

Step III $\frac{3}{4} = \frac{9}{12}$
$\frac{2}{3} = \frac{8}{12}$

Change to equivalent fractions having the denominator 12.

Step IV $\frac{17}{12}$

Add, as before.

Step V $\frac{17}{12} = 1\frac{5}{12}$ Answer

Change improper fraction to proper fraction in simplest form.

Job Assignment 18

Add the following fractions. Give answers in simplest form.

(1) $\frac{1}{3} + \frac{1}{4} =$ (Ans.)

(2) $\frac{3}{4} + \frac{6}{8} =$

(3) $\frac{2}{3} + \frac{1}{6} =$

(4) $\frac{5}{8} + \frac{1}{4} =$

(5) $\frac{1}{4} + \frac{7}{8} =$

(6) $\frac{1}{3} + \frac{3}{4} =$

(7) $\frac{1}{4} + \frac{1}{8} + \frac{3}{8} =$

(8) $\frac{2}{5} + \frac{7}{10} + \frac{4}{5} =$

(9) $6\frac{1}{3} + \frac{1}{4}$

(10) $2\frac{3}{8} + \frac{1}{4}$

Often we have mixed numbers to add. The procedure is shown in these examples:

Example A: Add 5 2/3 and 4 1/6.

Step I	$5\frac{2}{3}$ $+ 4\frac{1}{6}$	Write the problem.
Step II	Common Denom. = 6	Decide by testing 3 against 6 that 6 can be the common denominator.
Step III	$5\frac{2}{3} = 5\frac{4}{6}$ $4\frac{1}{6} = 4\frac{1}{6}$	Rewrite the mixed numbers using the common denominator of 6.
Step IV	$9\frac{5}{6}$ Answer	Add the whole-number portion. Add the fraction portion.

Example B: Add 6 1/3 and 3 3/4.

Step I	$6\frac{1}{3}$ $+ 3\frac{3}{4}$	Write the problem.
Step II	Common Denom. = 12	Decide that 12 is the common denominator.
Step III	$6\frac{1}{3} = 6\frac{4}{12}$ $3\frac{3}{4} = 3\frac{9}{12}$	Rewrite the mixed numbers using the common denominator of 12.
Step IV	$9\frac{13}{12}$	Add the whole-number portion. Add the fraction portion.
Step V	$9 + \frac{13}{12} = 9 + 1\frac{1}{12}$	Change the improper fraction to a mixed number, to be combined with the whole number.
Step VI	$10\frac{1}{12}$	Combine the two numbers.

Job Assignment 19

Add the following:

(1) $8\frac{1}{6} + 1\frac{1}{2} =$ (Ans.)

(2) $3\frac{5}{8} + 2\frac{1}{4} =$

(3) $3\frac{1}{2} + 7\frac{1}{3} =$

(4) $1\frac{1}{4} + 4\frac{1}{3} =$

(5) $4\frac{1}{2} + 6\frac{7}{8} =$

(6) $9\frac{5}{8} + 1\frac{3}{4}$

(7) $4\frac{1}{4} + 7\frac{2}{3} =$

(8) $5\frac{1}{5} + 7\frac{1}{2} + 9\frac{1}{10} =$

(9) $5\frac{3}{8} + 2\frac{1}{2} + 1\frac{3}{4} =$

(10) $6\frac{1}{8} + 3\frac{7}{8} + 1\frac{5}{8} =$

(11) $3\frac{1}{6} + 1\frac{1}{6} + 4\frac{1}{6} =$

(12) $7\frac{1}{2} + 1\frac{2}{3} + 1\frac{1}{6} =$

(13) $2\frac{3}{10} + 2\frac{2}{5} + 6\frac{1}{2} =$

(14) $5\frac{1}{2} + 9\frac{5}{16} + 1\frac{3}{4} =$

(15) $18\frac{1}{2} + 14\frac{3}{4} + 5\frac{7}{8} =$

(16) A recipe for buttercream icing calls for 2½ lbs. shortening, 1¾ lbs. egg whites, 10 lbs. powdered sugar, and 1 oz. (1/16 lb.) each of salt and flavoring. Find the total amount of icing this will produce.

Job Assignment 20

You are a front-end assistant in a supermarket and you have to pay your part-time employees. Below is a list of their hours. Figure out each employee's total hours.

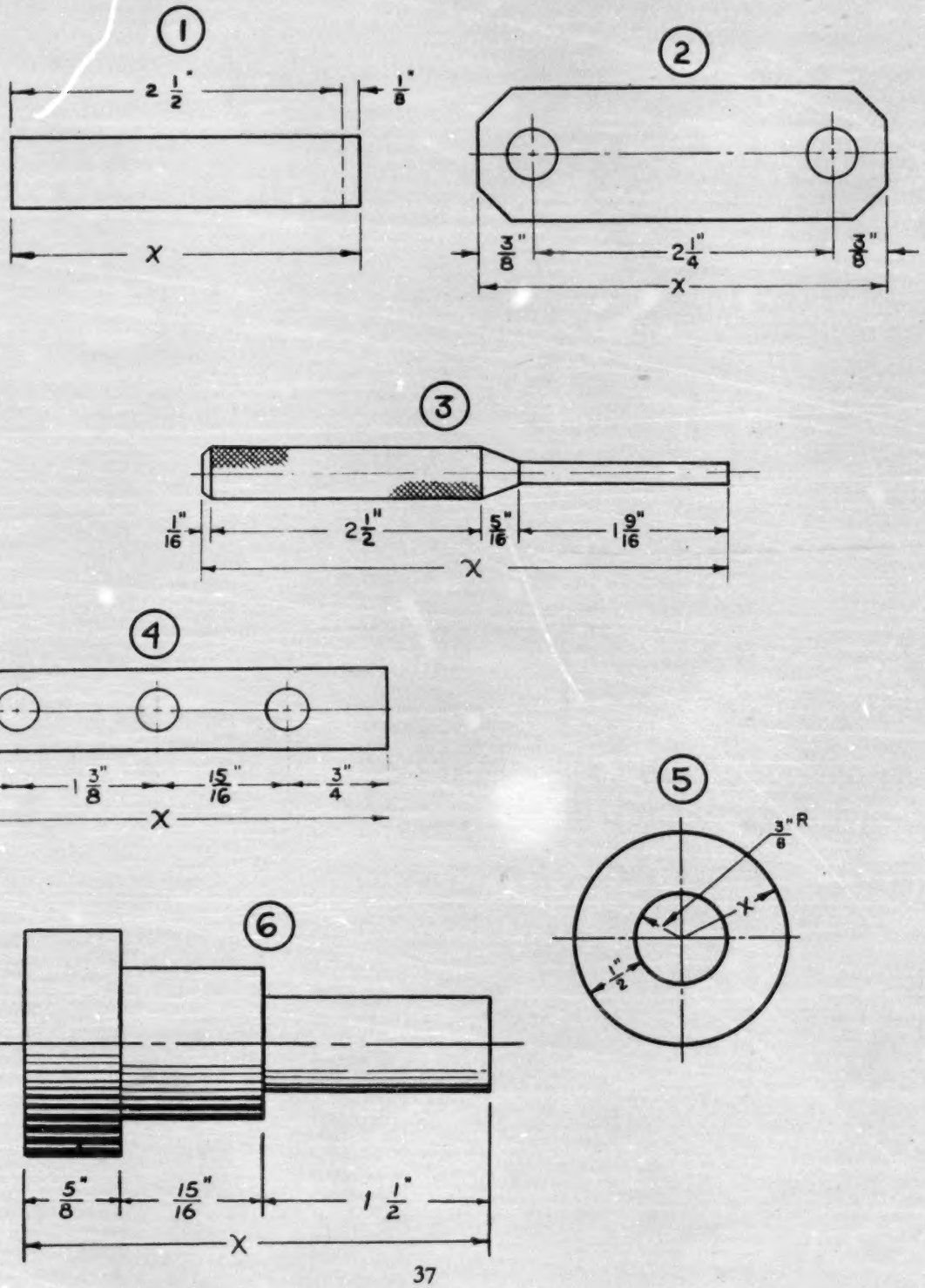
Time Record

Week Ending Sept. 30, 19___

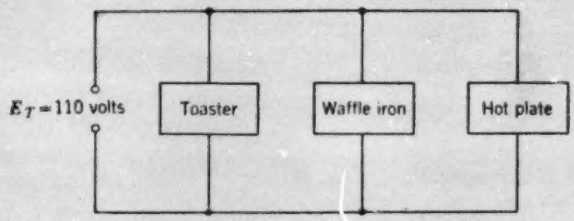
Employee	Mon.	Tues.	Wed.	Thurs.	Fri.	Total
R. Andrews	3	4½	4¼	3	3	
J. Billings	4	3¼	3¼	3	2½	
W. Dali	3¼	4½	4½	4	3¼	
E. Milano	2¾	4½	4	3¾	4¼	
H. McCann	4½	4	3	4½	4¾	
F. Wilkins	3¾	4	3	3¾	4	

Job Assignment 21

A machinist needs to be able to add and subtract fractions accurately. Find the missing dimensions in the following drawings. (Each is indicated by an X.)



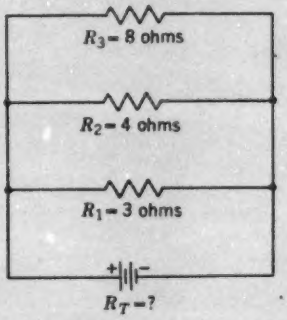
Job Assignment 22



Here is a diagram of three electrical resistors connected in parallel. A parallel circuit is one where the current divides into different paths.

Resistors tend to reduce the electrical flow. Resistance is measured in ohms. The total resistance R_T of a parallel circuit is given by the formula:

$$\frac{1}{R_T} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}$$



In the schematic diagram shown here,

$$\frac{1}{R_T} = \frac{1}{3} + \frac{1}{4} + \frac{1}{8}$$

Let us find R_T

The common denominator for the three resistances is 24. Rewriting the three fractions with the denominator of 24, we get:

$$\frac{1}{R_T} = \frac{8}{24} + \frac{6}{24} + \frac{3}{24} + = \frac{8 + 6 + 3}{24}$$

$$\frac{1}{R_T} = \frac{17}{24}$$

To get R_T, we must invert the fraction

$$R_T = \frac{24}{17} = 1\frac{7}{17} \text{ ohms}$$

In the chart below, find $1/R_T$ and R_T for the following circuits.

	Resistance				
1	2	3	4	$\frac{1}{R_T}$	R_T
(a) 5 ohms	10 ohms				
(b) 12 ohms	30 ohms	60 ohms			
(c) 50 ohms	10 ohms	50 ohms	5 ohms		
(d) 10 ohms	20 ohms	4 ohms			
(e) 12 ohms	13 ohms	4 ohms	6 ohms		

VII SUBTRACTING FRACTIONS

Once you have the knack of adding fractions, subtracting fractions becomes easy.

To subtract fractions having the same denominator:

1. Subtract the numerators.
2. Place the difference over the denominator.
3. Reduce the fraction where possible.

To subtract fractions having different denominators:

1. Find a common denominator.
2. Change the fractions to equivalent fractions having the new denominator.
3. Subtract numerators only.
4. Place the difference over the denominator.
5. Reduce the answer if possible.

Example A: 5 7/8 – 2 3/8.

Step I $5\frac{7}{8}$ − $2\frac{3}{8}$ — Write the problem. Note that the denominators are the same.

Step II $3\frac{4}{8}$ — Subtract numerators: 7 – 3 = 4; then place this difference over the denominator 8. Subtract the whole numbers.

Step III $3\frac{4}{8} = 3\frac{1}{2}$ (Ans.) — Reduce 4/8 to lowest terms.

Example B: 7 3/4 – 1 1/2

Step I $7\frac{3}{4}$ − $1\frac{1}{2}$ — Write the problem. Note that you will have to find a common denominator.

Step II Common Denom. = 4 — By testing 2 against 4, decide that 4 is the common denominator.

Step III $7\frac{3}{4} = 7\frac{3}{4}$; $-1\frac{1}{2} = 1\frac{2}{4}$ — Change to equivalent fractions with a denominator of 4.

Step IV $6\frac{1}{4}$ — Subtract numerators. Place the difference over the denominators. Subtract the whole numbers.

Job Assignment 23

Subtract:

(1) $\frac{7}{8} - \frac{5}{8} =$ (Ans.)

(2) $\frac{3}{4} - \frac{1}{8} =$

(3) $6\frac{5}{8} - \frac{3}{8} =$

(4) $4\frac{7}{10} - 1\frac{7}{10} =$

(5) $3\frac{3}{4} - 3\frac{1}{4} =$

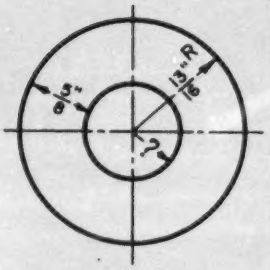
(6) $5\frac{3}{4} - 5\frac{1}{2} =$

(7) $10\frac{2}{3} - 6\frac{1}{4} =$

(8) $4\frac{1}{5} - 4\frac{1}{10} =$

(9) $7\frac{7}{8} - 2\frac{1}{2} =$

(10) Find the missing dimension of the washer shown here.



"Borrowing"

Up to now you have had to subtract smaller from larger fractions, which proved to be not too difficult. But suppose you had this problem:

$$6\frac{1}{8}$$
$$-\ 4\frac{7}{8}$$

How would you subtract the 7/8 from the 1/8?

When you practiced subtracting *whole numbers*, you often had to "borrow" 1 from the next place and put it in front of the number that had to be made larger. (See example here.) In effect you were borrowing *10*

$$\begin{array}{r} 75 \\ -\ 36 \\ \hline \end{array} \qquad \begin{array}{r} {}^{6}\cancel{7}{}^{1}5 \\ -\ 3\ 6 \\ \hline 3\ 9 \end{array}$$

from 70 (giving 60) and adding the 10 to the 5 to make 15. Then you were able to proceed with the problem.

We do something rather similar when we borrow in order to subtract fractions. This time we borrow 1 from the units place and add it to the fraction. (See example here.) We change the 6 to 5, and

$$\begin{array}{r} 6\frac{1}{8} \\ -\ 4\frac{7}{8} \\ \hline \end{array} \qquad \begin{array}{r} {}^{5}\not{6}\,1\frac{1}{8} \\ 4\ \ \frac{7}{8} \\ \hline \end{array}$$

we now have a 1 to add to our $\frac{1}{8}$ so that we can subtract. Our problem now becomes to subtract:

$1\frac{1}{8} - \frac{7}{8}$

If we changed $1\frac{1}{8}$ to an improper fraction, we could do it. We would have: $\frac{9}{8} - \frac{7}{8}$, which of course equals $\frac{2}{8}$ or $\frac{1}{4}$. Then $5 - 4 = 1$, and our answer is $1\frac{1}{4}$.

Let us summarize what we did. We will express it just a little differently this time.

$6\frac{1}{8}$ becomes $5\frac{8}{8}+\frac{1}{8}$ or $5\frac{9}{8}$

We can now subtract any proper fractionfrom $\frac{9}{8}$

Again, suppose we had to subtract:

$$7\frac{5}{12} - 3\frac{11}{12}$$

This time we would borrow 1 from the 7 and add $\frac{12}{12}$ to the $\frac{5}{12}$, giving $\frac{17}{12}$. Since this is an improper fraction (more than 1), any proper fraction (less than 1) can be subtracted from it.

Example A: Subtract 8 1/3 - 4 2/3

Step I
$$\begin{array}{r} 8\frac{1}{3} \\ -\ 4\frac{2}{3} \\ \hline \end{array}$$

Write the problem. Note that the denominators are the same but that 2/3 is larger than 1/3. You will have to borrow.

Step II
$$\begin{array}{rcl} 8\frac{1}{3} & = & 7\frac{3}{3} + \frac{1}{3} \\ -\ 4\frac{2}{3} & = & 4\frac{2}{3} \\ \hline \end{array}$$

Borrow 1 from 8, making it 7. Change the borrowed 1 to $\frac{3}{3}$ and add it to $\frac{1}{3}$.

Step III
$$\begin{array}{r} 7\frac{4}{3} \\ -\ 4\frac{2}{3} \\ \hline \end{array}$$

Adding $\frac{3}{3}$ to $\frac{1}{3}$ gives $\frac{4}{3}$

Step IV $3\frac{2}{3}$

Subtract fractions and subtract whole numbers.

Example B: 25 2/10 - 16 3/5

Step I $25\frac{2}{10}$ − $16\frac{3}{5}$ — Write the problem. Note that you will need to find a common denominator.

Step II Common Denom. = 10 — By testing 5 against 10, decide that 10 will be your common denominator.

Step III $25\frac{2}{10} = 25\frac{2}{10}$ − $16\frac{3}{5} = 16\frac{6}{10}$ — Change fractions to equivalent fractions, with a denominator of 10. Note that you will have to borrow.

Step IV $25\frac{2}{10} = 24\frac{10}{10} + \frac{2}{10}$ − $16\frac{6}{10} = 16\frac{6}{10}$ — Borrow 1 from 25, making it 24. Change the 1 to $\frac{10}{10}$ and add it to $\frac{2}{10}$

Step V $24\frac{12}{10}$ − $16\frac{6}{10}$ — Adding $\frac{10}{10}$ to $\frac{2}{10}$ gives $\frac{12}{10}$

Step VI $8\frac{6}{10} = 8\frac{3}{5}$ — Subtract fractions and whole numbers.

Job Assignment 24

Subtract the following. Reduce answers to lowest terms.

(1) $6\frac{1}{4} - 4\frac{1}{4} =$ (Ans.)

(2) $2\frac{1}{3} - 1\frac{1}{3} =$

(3) $16\frac{9}{10} - 5\frac{8}{10} =$

(4) $6\frac{2}{3} - 3\frac{1}{3} =$

(5) $5\frac{1}{4} - 2\frac{1}{2} =$

(6) $7\frac{1}{2} - 4\frac{4}{5} =$

(7) $7\frac{5}{12} - 2\frac{5}{6} =$

(8) $8\frac{5}{6} - 4\frac{2}{3} =$

(9) $16\frac{1}{10} - 3\frac{3}{5} =$

(10) $7\frac{3}{4} - 6\frac{1}{6} =$

(11) $5\frac{1}{4} - 1\frac{2}{3} =$

(12) $9\frac{1}{4} - 5\frac{4}{5} =$

(13) $7\frac{1}{3} - 1\frac{3}{4} =$

(14) $2\frac{3}{8} - 1\frac{5}{16} =$

(15) $3 - 1\frac{1}{8} =$

(16) $7\frac{1}{4} - 5\frac{3}{16} =$

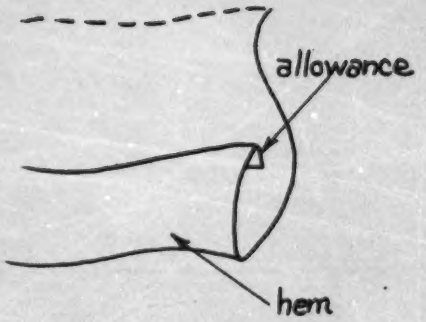
(17) $8\frac{3}{8} - 3\frac{3}{4} =$

(18) $15 - 4\frac{1}{4} =$

(19) $6\frac{5}{12} - 5\frac{2}{3} =$

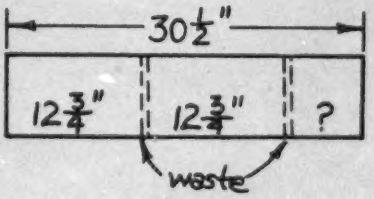
(20) $14\frac{3}{16} - 13\frac{3}{4} =$

Job Assignment 25



(1) A dressmaker had $3\frac{3}{8}$ inches of fabric around the bottom of a skirt to use for the hem. If she needed a ¼-inch allowance to turn under, how wide would the hem itself be?

(2) The cook bought 18½ lbs. of chicken. Of this, 4¾ pounds was waste – bones, skin, etc. How much chicken meat was usable?



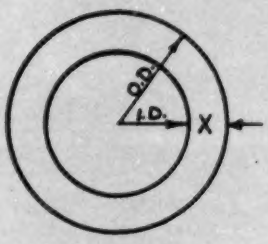
(3) Brain had to cut two boards 12¾ inches long out of a piece of wood 30½ inches long. If each cut wasted 1/16 inch, how big a piece would be left over?

(4) A draftsman had to make a drawing of the following lengths: 1 3/8", 3 5/8", and 4 3/4", to be placed next to each other. Will his 10" paper be long enough?

(5) How much has to be cut off a piece of metal stock that is 3 1/16 inches thick to get it down to 2 15/16 inches?

(6) Stella's pattern called for 3 3/4 yards of fabric. The salesperson pointed out that she had a nice remnant on sale that was 4 1/3 yards long. If Stella bought the remnant, how much fabric would be left over?

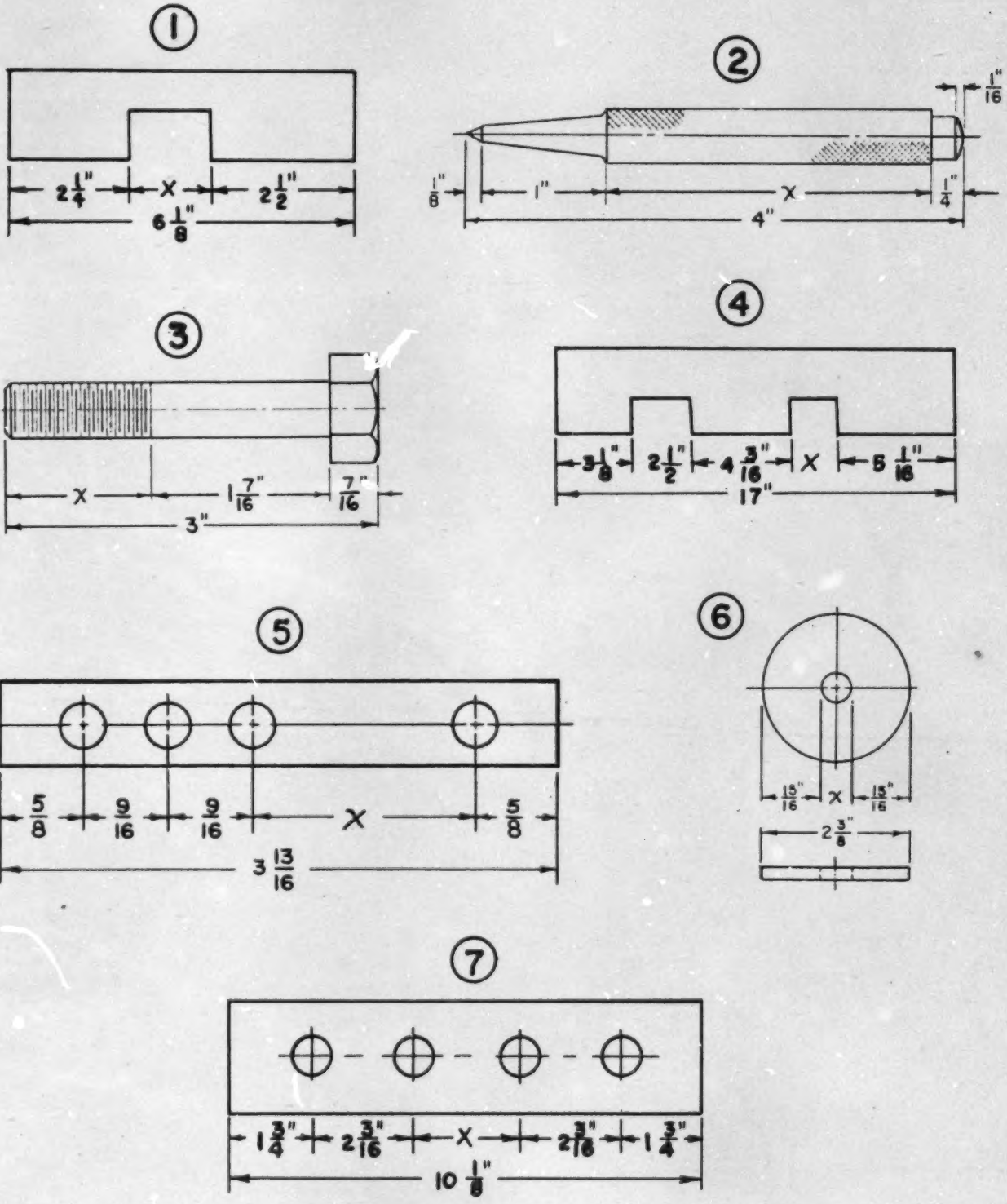
A series of washers has the following measurements. Find X in each case.



	O.D.	I.D.	X
7.	1 7/8"	1 1/8"	
8.	2"	3/4"	
9.	1 15/16"	1 1/4"	
10.	2 1/4"	1 3/8"	

Job Assignment 26

Find the missing dimension (X) in the drawings below.



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VIII – REVIEW EXERCISE #1

Add the following. Reduce all answers.

(1) $1\frac{1}{2}$ + $3\frac{2}{3}$

(2) $3\frac{1}{4}$ + $7\frac{1}{3}$

(3) $4\frac{1}{2}$ + $2\frac{11}{12}$ + $6\frac{2}{3}$

(4) $9\frac{5}{8}$ + $1\frac{1}{4}$ + $\frac{7}{16}$

(5) $5\frac{4}{5}$ + $\frac{1}{2}$ + $9\frac{7}{10}$

(6) $2\frac{3}{10}$ + $2\frac{2}{5}$ + $6\frac{1}{2}$

Subtract the following. Reduce all answers.

(7) $6\frac{3}{4} - 5\frac{1}{4}$

(8) $7\frac{3}{8} - 6\frac{3}{4}$

(9) $8\frac{3}{10} - 4\frac{1}{5}$

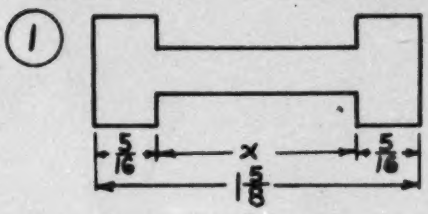
(10) $6\frac{1}{12} - \frac{1}{8}$

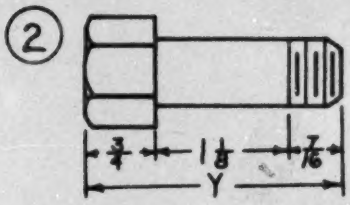
(11) $15 - 4\frac{3}{10}$

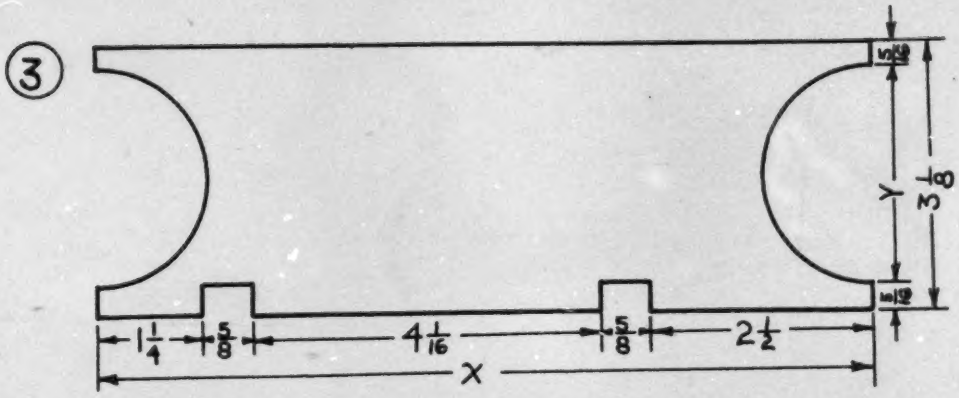
(12) $6\frac{2}{3} - 3\frac{3}{4}$

REVIEW EXERCISE #2

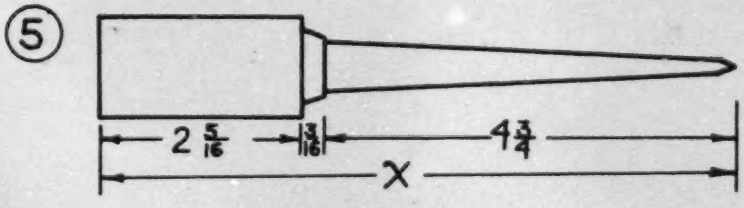
Find the missing dimensions

1. 

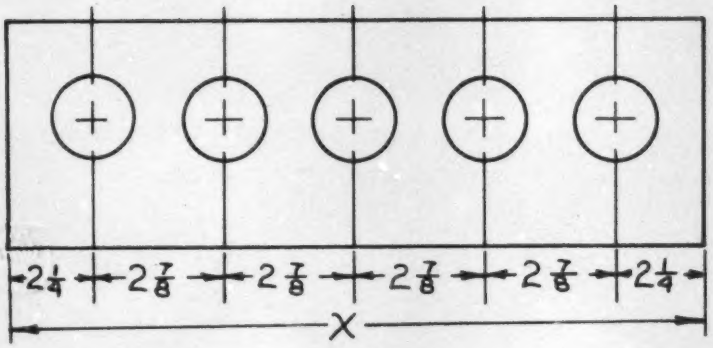
2. 

3. 

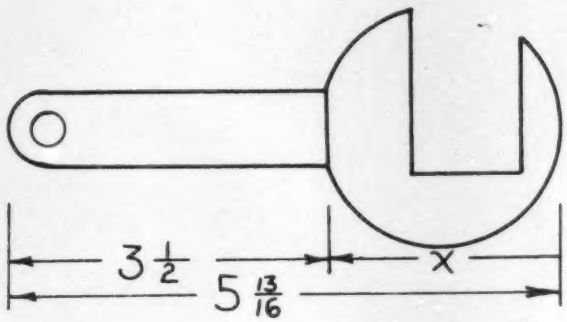
4. In problem #3, find X.

5. 

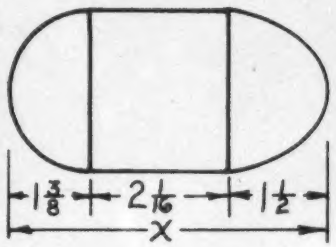
⑥



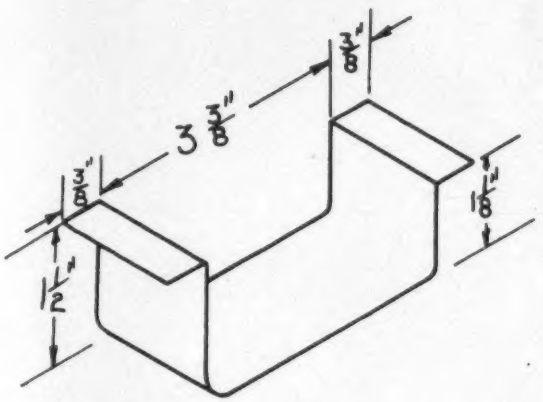
⑦



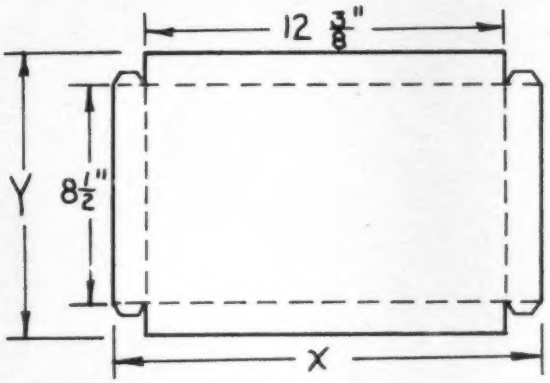
⑧

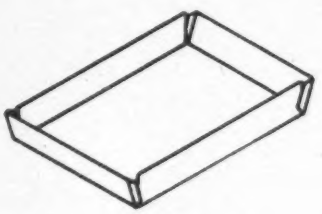


⑨ Find the total length of sheet metal needed for the hanger shown at the right.



⑩ To make a sheet metal pan, the pattern shown below was made. If the upright sides are to be 1¾" high, how large a piece of sheet metal must be used?





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QUIZ ON FRACTIONS

Do the indicated operations. Reduce all answers.

(1) $\frac{2}{3} + \frac{3}{4} + \frac{11}{12}$ _____

(2) $6\frac{7}{8} - 6\frac{3}{8}$ _____

(3) $3\frac{1}{4} \div 4$ _____

(4) $\frac{3}{8} \times 1\frac{1}{2}$ _____

(5) $15\frac{1}{2} + 22\frac{9}{16} + 10\frac{5}{8}$ _____

(6) $2\frac{5}{16} + \frac{3}{8} + 4\frac{3}{4}$ _____

(7) $1\frac{5}{8} - \frac{11}{16}$ _____

(8) $17 - 12\frac{7}{8}$ _____

(9) $6\frac{2}{3} \times 2\frac{1}{2}$ _____

(10) $\frac{5}{8} \div \frac{1}{4}$ _____

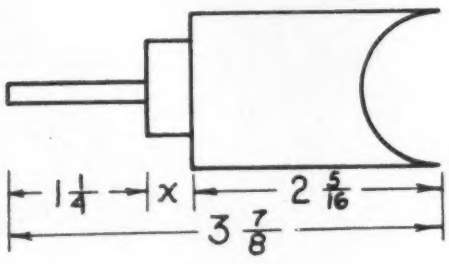
(11) $6\frac{1}{12} - 1\frac{1}{8}$ _____

(12) $4\frac{1}{5} \div 7$ _____

(13) $\frac{1}{2} \times 2\frac{3}{4}$ _____

Find the missing dimension

(14)



(15)